In order to find the dependence of nonlinearity parameter for water-saturated andesite on effective pressure on the basis of relationship (1.8) the following data were used: K_2 = 2.62 GPa, K_1 = 48.8 GPa, $\rho_2/\rho_1 = 0.4$, $\varepsilon_2 = 4$. Since in the pressure range in question equation of state (1.5) for the solid phase may be linearized, in the calculations it was assumed that $\varepsilon_1 = 1$.

As follows from Fig. 5, the value of ε with small p_{eff} for strongly cemented rocks may reach the order of 10^2. In contrast to weakly cemented rocks the nonlinearity parameter for material with $K/K_1 \sim 1$ depends weakly on water saturation. It can be seen from the calculations that attenuation caused by interphase friction is markedly less than that observed by experiment for strongly cemented rocks.

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SOLUTION OF A NONSTATIONARY PROBLEM OF ELASTICITY THEORY

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UDC 539.3

In this paper we present a new approach to the solution of nonstationary anti-plane boundary value problems of linear elasticity theory for semi-bounded regions of the type of a halfspace or a layer with mixed boundary conditions both on their surfaces (systems of stamps) and also their interiors (cracks, inclusions). Application of an additional integral Laplace transform with respect to the time for reducing the above-named boundary value problems to the solution of an integral equation gives rise to certain difficulties in its solution in comparison with problems of stationary oscillations, methods for the solution of which are, at the present time, well worked-out. The majority of processes, however, are essentially of a nonstationary nature and cannot be reduced to problems of harmonic analysis. The solution, therefore, of nonstationary problems calls for urgent attention.

According to the method we propose, using properties of the inversion of Laplace and Fourier convolutions of two functions, the initial boundary value problem can be reduced to the solution of a Volterra integral equation of the first kind for the unknown function itself and not its integral transform. In this connection, the Laplace and Fourier transforms are carried over with the unknown function onto the kernel, which is given by analytic expression in explicit form. The original of this kernel is then found by Cagniard's method

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as modified by De Hoop [1]. Solution of the resulting integral equation is carried out by numerical methods based on a discretization of the time interval and an expansion of the unknown function on each such elementary interval on a three-dimensional basis of finite functions. We illustrate the method by solving an anti-plane nonstationary problem involving a stamp on the surface of an elastic halfspace. Numerical results are presented.

In contrast to numerical methods for solving boundary value problems, our approach enables us to investigate not only quantitative, but also qualitative, characteristics of a solution. On the other hand, in contrast to the presently favored asymptotic methods for solving integral equations involving the Laplace-transformed unknown function and its subsequent numerical inversion, here the Laplace and Fourier inversions are obtained from the unknown function (appearing in the kernel of the integral equation) exactly, and not approximately, with the aid of Cagniard's method, which, undoubtedly, increases the accuracy of the solution and simplifies its computational aspects.

We consider oscillations of the surface of an elastic halfspace $z_0 \le 0$, $|x_0, y_0| < \infty$, due to short-term shear displacements of a stamp $V_0(x_0, t_0)$, $|x_0| \le a$, $0 \le t_0 \le T$, present on the surface of this halfspace. Displacements $v_0(x_0, y_0, t_0)$ of the halfspace itself along the OY-axis satisfy the wave equation

$$\left(\frac{\partial^2}{\partial x_0^2} + \frac{\partial^2}{\partial z_0^2}\right) v_0(x_0, z_0, t_0) = \frac{1}{b^2}\frac{\partial^2}{\partial t_0^2} v_0(x_0, z_0, t_0) \tag{1}$$

(b is the speed of transverse waves). On the surface of the halfspace we are given the mixed boundary conditions

$$\begin{gathered} v_0(x_0, 0, t_0) = V_0(x_0, t_0),\ z_0 = 0,\ |x_0| \leqslant a, \\ \tau_{zy}^0(x_0, 0, t_0) = 0,\ z_0 = 0,\ |x_0| > a, \end{gathered} \tag{2}$$

while infinity conditions for decrease of the amplitude of displacements are satisfied. For simplicity we assume zero initial conditions.

Application of Laplace and Fourier integral transforms to Eqs. (1) and (2) with subsequent use of properties of the convolution of two functions makes it possible to reduce the initial boundary value problem to the solution, in dimensionless variables, of the integral equation

$$\int_0^t dT \int_{-1}^{1} \tau(\xi, T)\, k(x-\xi, t-T)\, d\xi = V(x, t),\ |x| \leqslant 1, t \geqslant 0 \tag{3}$$

relative to the unknown contact stresses $\tau(\xi, T)$ under the stamp. Here $V(x, t) = \pi V_0(x, t)/a$; $x = x_0/a$; $t = t_0 b/a$; $\tau(\xi, T) = \tau_{zy}{}^0(\xi, T)/\mu$; μ is the shear modulus of the material comprising the halfspace.

The kernel of the integral equation (3),

$$k(x-\xi, t-T) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\alpha(x-\xi)}\, d\alpha\, \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{1}{\sqrt{\alpha^2+s^2}} e^{s(t-T)}\, ds \tag{4}$$

in this case can be calculated with the aid of integral transform tables [2]. For more involved functions it is necessary to apply Cagniard's method [3], the essence of which is that by deforming the integration contour the inverse Fourier transform is brought to the form of a direction Laplace transform, thereby making it possible to obtain directly from expression (4) the original of the kernel itself. As a result of Eq. (3), we arrive at the equation

$$\int_0^t dT \int_{-1}^{1} \tau(\xi, T)\frac{H(t-T-|x-\xi|)}{\sqrt{(t-T)^2-(x-\xi)^2}}\, d\xi = V(x, t),\ |x| \leqslant 1, t \geqslant 0 \tag{5}$$

[H(y) is the Heaviside function].

Dimensionless displacements of interior points of the halfspace may be expressed in a similar way in terms of the contact stresses:

$$v(x, z, t) = \int_0^t dT \int_{-1}^{1} \tau(\xi, T) \frac{H(t - T + z) H\left(\sqrt{(t-T)^2 - z^2} - |x - \xi|\right)}{\sqrt{(t-T)^2 - z^2 - (x-\xi)^2}} d\xi,$$
$$|x| < \infty, \ z \leqslant 0, \ t \geqslant 0.$$

Continuation of the right-hand side of the integral equation (5) beyond the boundary of the region occupied by the stamp defines a wave field on the surface of the halfspace generated by movements of the stamp. Following the picture of the wave front in the domain $|x| \geq 1$, $z = 0$, we conclude from the analytical representation of the kernel in Eq. (5) that at points with coordinates $x > 1$ the free surface of the halfspace is in a state of quiescence up to the time (dimensionless) $t = x - 1$, following which a wave from the stamp arrives there. The coordinates $x = t + 1$ and $x = -t - 1$ determine the forward front of the wave to the right and to the left of the stamp, respectively. Similar statements can be made concerning the wave field inside the halfspace.

To solve Eq. (5) we use the method of discretization with respect to the time and expand the unknown function $\tau(x, t)$ in a basis of finite functions. A scheme of this kind was used in [4] for the solution of boundary value problems by the boundary element method. Here the time interval of integration $[0, t]$ is partitioned by the points t_m, $(m = 0, 1, \ldots, M)$, into M parts with time step Δt ($t_0 = 0, \ldots, t_M = t$), and the spatial interval of integration $[-1, 1]$ is partitioned by the points x_n ($n = 0, 1, \ldots, N$) into N parts with step size h ($x_0 = -1, \ldots, x_N = 1$). Taking into account a root singularity in the stresses $\tau(x, t)$ at the edges of the stamp [5], we expand the unknown contact stresses on each time interval $[t_{m-1}, t_m]$ in a series

$$\tau_m(x) = \sum_{n=0}^{N} C_{m,n} \psi_n(x), \quad m = 1, 2, \ldots, M \tag{6}$$

according to the following basis:

$$\psi_0(\xi) = \frac{x_1 - \xi}{h\sqrt{\xi + 1}}, \quad -1 \leqslant \xi \leqslant x_1, \ \psi_N(\xi) = \frac{\xi - x_{N-1}}{h\sqrt{1 - \xi}}, \quad x_{N-1} \leqslant \xi \leqslant 1,$$
$$\psi_n(\xi) = \begin{cases} \dfrac{\xi - x_{n-1}}{h}, & x_{n-1} \leqslant \xi \leqslant x_n, \\ \dfrac{x_{n+1} - \xi}{h}, & x_n < \xi \leqslant x_{n+1}, \\ 0, \ \xi < x_{n-1}, \ \xi > x_{n+1}, & n = 1, 2, \ldots, N - 1. \end{cases}$$

The coefficients $C_{m,n}$ in Eq. (6) are different on each time interval indicated, and the values of $\tau_m(x)$ are themselves constants with respect to the time and denote values of the contact stresses on this time interval.

As a result, Eq. (5) acquires the form

$$\sum_{n=0}^{N} C_{M,n} \int_{t_{M-1}}^{t} dT \int_{-1}^{1} \frac{H(t - T - |x - \xi|)}{\sqrt{(t-T)^2 - (x-\xi)^2}} \psi_n(\xi) d\xi = V(x, t) -$$
$$- \sum_{m=1}^{M-1} \sum_{n=0}^{N} C_{m,n} \int_{t_{m-1}}^{t_m} dT \int_{-1}^{1} \frac{H(t - T - |x - \xi|)}{\sqrt{(t-T)^2 - (x-\xi)^2}} \psi_n(\xi) d\xi, \quad |x| \leqslant 1, t \geqslant 0, \tag{7}$$

where $C_{M,n}$ are coefficients in the expansion of the unknown contact stress at a given time t relative to the basis $\psi_n(x)$; these coefficients can be found from an algebraic system obtainable from Eq. (7) by successively putting $x = x_n$ ($n = 0, 1, \ldots, N$). Obviously, for this it is necessary to know the coefficients $C_{m,n}$ in the expansion of the contact stresses in the series (6) on the previous time intervals $m = 1, 2, \ldots, M - 1$. In the resulting recursion formula we have, at the first step, a very simple algebraic system

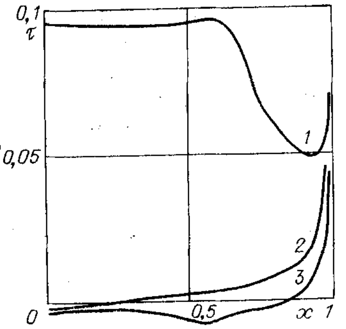

Fig. 1

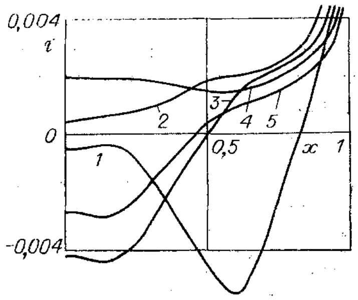

Fig. 2

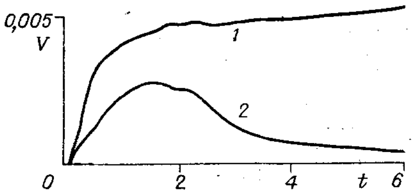

Fig. 3

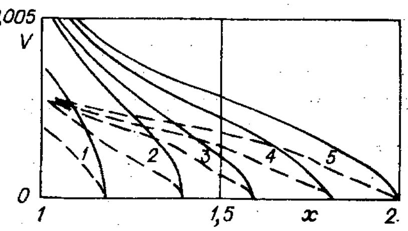

Fig. 4

$$\sum_{n=0}^{N} C_{1,n}\int_0^{t_1} dT \int_{-1}^{1} \frac{H(t_1 - T - |x_k-\xi|)}{\sqrt{(t_1-T)^2-(x_k-\xi)^2}}\,\psi_n(\xi)\,d\xi = V(x_k,t_1),\quad k=0,1,\ldots,N$$

for the unknown $C_{1,n}$.

The coefficients of this and also of all the subsequent systems are double integrals of the unknown functions. Integrals of $\psi_n(x)$ ($n = 1, 2, \ldots, N - 1$) may be calculated analytically, while those of $\psi_0(x)$ and $\psi_N(x)$ may be reduced to elliptic integrals and handled numerically. Matrices of all the systems are well defined and have elements only on the principal diagonal and on the two diagonals closest to it. According to the estimates given in [4], the convergence of this method is guaranteed providing $\Delta t/h^2 \leq 0.5$.

Computer programs were written and a numerical analysis was made of the wave fields on the surface of the halfspace and of the contact stresses under the stamp. Two cases were considered: 1) the stamp executes a small movement according to the law $V(x, t) = 0.3t$ over the dimensionless time interval $0 \leq t \leq 0.04$ and is then fixed in this displaced position; 2) under these same conditions, after the time $t = 0.04$ the stamp becomes disengaged from the surface of the halfspace and remains free of it for all further time.

Figures 1 and 2 exhibit graphs of the dimensionless contact stresses for Case 1 for $t_1 = 0.04$, $t_2 = 0.08$, $t_3 = 0.4$, and for $t_1 = 0.4$, $t_2 = 0.6$, $t_3 = 0.8$, $t_4 = 1$, $t_5 = 1.2$, respectively. The curves are numbered to correspond to graphs of the stresses at the times t_n. By virtue of symmetry the graphs are given only for one half of the stamp, $0 \leq x \leq 1$.

In Fig. 3 graphs 1 and 2, corresponding to Cases 1 and 2, show the variation with time of the dimensionless displacement of a point of the free surface of the halfspace 0.2 units distant from the edge of the stamp. For $t < 0.2$ the forward front of the wave has as yet not reached the point in question. When $t = 0.2$ a wave from the displacement of the closest point of the stamp arrives at this point, and when $t = 2.2$ a wave arrives from the farthest point of the stamp. For times t sufficiently large a displacement of the point of the surface in question tends towards its displacement under a static load in Case 1; in Case 2 it tends towards zero.

Figure 4 presents graphs of dimensionless displacements of points of the free surface of the halfspace as a function of the distance x from the edge of the stamp ($x = 1$) for $t_1 = 0.4$, $t_2 = 0.8$, $t_3 = 1.2$, $t_4 = 1.6$, and $t_5 = 2$ for Case 1 (solid curve) and Case 2 (dashed curve). Curves numbered 1 to 5 are the graphs of displacements of points of the surface of the halfspace at times t_n.

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CONJUGATE PROBLEM OF AERODYNAMIC EXTRUSION OF JETS OF HEATED VISCOUS LIQUID

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UDC 532.526

Aerodynamic extrusion of jets of viscous liquids is of practical value for the production of synthetic filaments consisting of polymer melts with the help of high-velocity gas flows. The problem of fiber formation is a conjugate problem, in which the mutual effect of the fiber formed and the surrounding medium must be taken into account. This problem was first formulated mathematically in [1, 2], where a model of the flow is proposed and the basic equations and boundary conditions are derived. In [3, 4] the most general equations describing the dynamics of thin jets of viscous liquid are derived taking into account the spatial bending and twisting, and in [4-6] the present status of the theory of hydrodynamics, heat transfer, and stability of fiber formation processes are analyzed in detail. In the case of fiber formation with the help of extrusion devices with low (up to 5 m/sec) final velocities of the jets, the external force exerted by the flow can be neglected [7]. As the velocity of the filament increases the effect of friction on the parameters of the fiber becomes significant. For aerodynamic extrusion the forces of interaction of the fiber and flow are determining. A number of works (for example, [8, 9]), in which the characteristics of aerodynamic formation are studied, are devoted to some physical and technological aspects of this problem. In this paper we construct a complete, conjugate mathematical model of the flow and we perform a numerical analysis based on an iteration method [10, 11].

1. Basic Equations and Boundary Conditions. Figure 1 shows a diagram of the flow of a jet of liquid, extruded with an air flow, parallel to the axis of the jet (1 – draw hole, 2 – jet, 3 – ejector). Because of the existence of viscous and heat-conduction effects, the jet of melt and the exterior medium interact with one another by means of the boundary layer. The mutual effect of the extruded jet and the medium makes this problem a conjugate problem. Let us assume that the flow of the jet of melt is stable, the jet does not bend and does not oscillate, and the velocity and temperature profiles in the jet are uniform. These assumptions make it possible to employ simple equations of motion of the liquid jet and heat transfer, derived, for example, in [1-3]:

$$\frac{dA_j}{dx}=\frac{\rho_j A_j F}{G\beta},\quad \frac{dT_j}{dx}=\frac{2\pi r_j q}{\rho_j u_j c_j A_j},$$
$$G=\rho_j u_j A_j,\ F=F_{fr}+F_{in}+F_g,\ q=q_T+q_{rad}\tag{1.1}$$
$$\beta=D\exp(B/T_j+C),\ A_j|_{x=0}=A_{j0},\ T_j|_{x=0}=T_{j0}.$$

Here ρ_j is the density of the liquid; u_j is the velocity of the jet; ε is the emissivity of the body; σ is the Stefan–Boltzmann constant; c_j is the heat capacity of the liquid; A_j is the area of the transverse cross section of the jet; T_j is the temperature of the jet; β is the longitudinal viscosity of the polymer; G is the flow rate of the polymer; r_j is the radius of the jet; F is the total axial force, balancing the rheological force and including

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